Protocol of F-Praktikum
Experiment
Laser

submitted by

Simon Wolfgang Mages and Florian Rappl

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Contents

1 Introduction

A LASER (**L**ight **A**mplification by **S**timulated **E**mission of **R**adiation) can provide monochromatic, coherent light in a very broad range of wavelengths and intensities with low beam divergence. Lasing is based on one of A. Einstein's famous papers about blackbody radiation. He introduced the concepts of absorption, spontaneous, and stimulated emission of photons by quantum systems. The latter is crucially important for laser activity. Therefore one needs population inversion between two energy levels in a quantum system (and therefore three or four levels and an (efficient) pumping mechanism). To achieve amplification of light one furthermore needs positive feedback meaning a resonator around the active medium with the quantum systems responsible for lasing.

Figure 1: The working He-Ne Laser

2 Theoretical framework

2.1 Quantum mechanical background

A quantum system of bound states has only discrete energy levels available. For simplicity we only consider two nondegenerate ones. If there are many of identical systems in thermal equilibrium the relative population of the levels is determined by the Maxwell-Boltzmann-distribution:

$$N_n = \exp\left(\frac{-E_n}{k_B T}\right),$$

where N_n is the population of the level with energy E_n, T is the absolute temperature, and k_B Boltzmann's constant. Let's take $E_2 > E_1$. Then we have

$$\frac{N_2}{N_1} = \exp\left(\frac{E_1 - E_2}{k_B T}\right) < 1 \quad \text{(no population inversion).}$$

Thermal equilibrium is a steady state meaning transitions from E_2 to E_1 must equal those in the other direction. Our systems are considered to interact by means of electromagnetic processes exclusively (only elastic collisions, etc.). Then the only possible way for a system to get from E_1 to E_2 is to absorb the energy difference

$$E_2 - E_1 =: \hbar\omega$$

in form of a photon of a radiation field, with ω the frequency of that photon and $\hbar$ Planck's constant. For the other direction there are two possibilities both of them liberating the energy difference as a photon: spontaneous emission and induced (stimulated) emission. The probabilities of these transitions are given by the Einstein coefficients. The probability of spontaneous emission

$$W_{21,sp} = A_{21,A}$$

is independent of external fields, the probabilities of stimulated emission

$$W_{21,st} = B_{21}\rho(\omega)$$

and of absobtion

$$W_{12} = B_{12}\rho(\omega)$$

are not, $\rho(\omega)$ being the energy density of the radiation field. In thermal equilibrium (where

$$N_1 W_{12} = N_2(W_{21,sp} + W_{21,st}),$$

number of absorbed photons = inclusive number of emitted photons, both per unit time) no light amplification takes place. Nevertheless by considering this case one can fix some of the constants:

$$B_{12} = B_{21}, \qquad A = \frac{\hbar\omega^3}{\pi^2 c^3} B_{21},$$

thus

$$W_{21} = W_{21,sp} + W_{21,st} = \left(\frac{\hbar\omega^3}{\pi^2 c^3} + \rho(\omega)\right) B_{21}.$$

The only amplifying process here is stimulated emission (see below). To achieve light amplification one has to increase the number of photons emitted by stimulated emission $N_2 W_{21,st}$ over the number of absorbed photons $N_1 W_{12}$, that is

$$\frac{N_2}{N_1} > \frac{W_{12}}{W_{21,st}} = 1.$$

Therefore Population inversion is needed. Additionally one gets the Lambert-Bouger law for intensity changes in the medium:

$$I(x) = I_0 \exp(-\alpha x), \qquad \alpha = (N_1 - N_2) B_{12} \hbar \frac{\omega}{c}.$$

2.2 A closer look on stimulated emission

Looking at the three possible transitions more closely one gets a very important aspect for laser operation: the process of spontaneous emission produces a photon like the induced emission, but in the latter case this photon has very well defined properties: it has the same energy, same phase, same polarization, and same direction of propagation like the photon which induced the emission, whereas the spontaneously emitted photon has the same energy, but is of randomly chosen polarization, direction of propagation, and phase relative to other photons in the system. The property of reproducing the phase of incident light leads to high temporal coherence, reproducing the direction of propagation leads to high spacial coherence. By selecting one direction of propagation using a resonator one gets these properties in laser light. By choosing a direction of polarization inside the resonator (e.g. by Brewster windows) one gets polarized laser light. The process of amplification by stimulated emission gives also rise to an increase of intensity, of course.

2.3 Resonator modes

Besides selecting a direction and providing positive feedback the resonator of a laser determines other properties of the laser beam, the transversal and longitudinal mode structure. The longitudinal modes are a simple result of the geometry of the resonator and interference of electromagnetic waves. Let's consider a resonator made up of two parallel mirrors. The resonator then acts like a Fabry-Perrot element: standing waves and thus non-zero fields are only possible if an integer multiple of the wavelength equals the optical way in the resonator, that is

$$\lambda = \frac{2L}{n},$$

where L is the (optical) resonator length and n is any positive integer. The transversal mode structure is the intensity distribution of the beam perpendicular to the beam axis. It is a result from the condition that the transverse field distribution in the resonator should remain unchanged after one roundtrip in the resonator in order not to interfere destructively. Therefore one seeks for solutions of the wave equation

$$\nabla^2 \vec{E}(\vec{r}, t) + \frac{1}{c^2} \frac{d^2}{dt^2} \vec{E}(\vec{r}, t) = 0,$$

which have a finite cross-section in order to minimize losses and changes of the field distribution caused by diffraction at the finite reflecting areas of the resonator mirrors. By adjusting the wave equation to our purposes, that is monochromatic solution

$$\vec{E}(\vec{r}, t) = \vec{E}(\vec{r}) \exp(i\omega t), \qquad k = \frac{\omega}{c},$$

beamlike solution: light travels close to z axis in z direction

$$\frac{d^2}{dz^2} \vec{E}(\vec{r}) \approx 0,$$

one gets the paraxial wave equation:

$$\frac{d^2}{dx^2} \vec{E}(\vec{r}) + \frac{d^2}{dy^2} \vec{E}(\vec{r}) - 2ik \frac{d}{dz} \vec{E}(\vec{r}) = 0.$$

Solution of this equation is the general Gaussion beam solution. It can be written in terms of rectangular or radially symmetric basis functions using Hermite- or Laguerre-polynomials to get Hermite-Gauß-beams or Laguerre-Gauß-beams. These solutions determine the transversal field and intensity structure and are therefore the transversal modes. Both sets of solutions share the property that their constituents can be labeled unambiguously by the number of nodes they have in each independent direction or coordinate, i.e. number of nodes in x- and y-direction for rectangular symmetry or in r- and ϕ-coordinate for spherical symmetry. Thus the solutions and transversal modes are labeled by TEM_{mn} for Transversal Electromagnetic Mode with integers m and n for the number of zeros of the field and intensity for both directions.

2.4 Resonator types

There are different types of resonators besides the Fabry-Perrot-like one with plane-parallel mirrors. They are classified by the radii of their mirrors (plane-parallel, spherical/concentric, confocal, hemispherical, and concave-convex). Stability of a resonator means that a slightly off-centered ray can't escape from the resonator after an infinite number of roundtrips. The condition for stability is often denoted by

$$0 \leq g_1 g_2 \leq 1, \qquad g_i = 1 - \frac{L}{R_i},$$

with L being the length of the resonator, R_i the radius of the mirror.

2.5 Concept of the HeNe-Laser

Main components

- capillary tube,
- 2 mirrors,
- 2 electrodes,
- 2 Brewster-windows.

Components of the capillary tube

- Contents up to approx. 85% Helium (pump-gas) and up to approx. 15% Neon (Lasergas),
- Diameter of approx. 1 mm,
- Length some 10 cm,

- Gas pressure approx. 100 Pa,
- integrated electrodes for gas discharge.

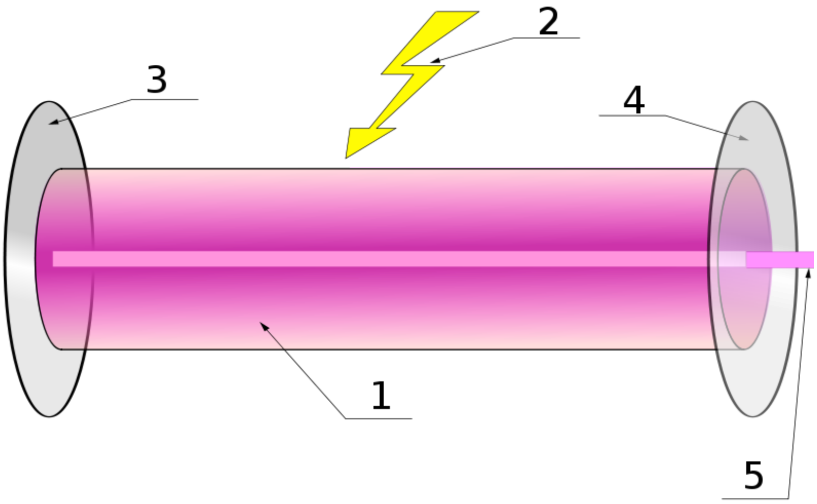

Figure 2: Principal building plan (from [1])

The captions in the figure above are standing for

1. the active medium (Helium-Neon gas mixture)
2. pumpenergy over voltage
3. plain mirror (Reflexion coefficiant approx. $99,9\%$)
4. semitransparent concave mirror (Transmission coefficiant approx. 1%)
5. parallel light ray

Remark A concave mirror is used in order to have more tolerance in the justification of the mirrors (else the mirrors must be exactly parallel) and to concentrate outgoing light to one parallel light ray.

2.6 Functionality of the HeNe-Laser

The physical reaction can be divided into four parts:

1. Gas discharge brings *He* atoms from the ground level to the first excited state,

$$E_1^{He} = E_0^{He} + 20.61eV.$$

   The relaxation time in this state is about $\tau = 10^{-3}s$.

2. *Ne* atoms are stimulated by collisions with the excited *He* atoms,

$$E_2^{Ne} = E_1^{He} + 0.05eV.$$

3. The neon atoms which emerge from the collisions will emit a photon and go to their first excited, metastable state which is normally not populated.

$$E_1^{Ne} = E_2^{Ne} - 1.96eV.$$

   Thus the population inversion takes place.

4. With the population inversion we get a photon avalanche and an enhancement of the electromagnetic waves. Through spontaneous emission we gather a photon with

$$\hbar\omega = 1.96eV, \qquad \lambda = 632.8 \text{ nm}.$$

3 Experiment

For performing the different tasks of this F-Praktikum experiment we used different devices. Additionally to the two built-in mirrors we needed to use two other mirrors, $R_1 = 1$ m, $R_2 = 0.75$ m. For measuring the intensity of the laser's beam we used a solar cell, as well as different (absorbating) glas plates. A detailled description of the devices we used for each task can be found in the task's section.

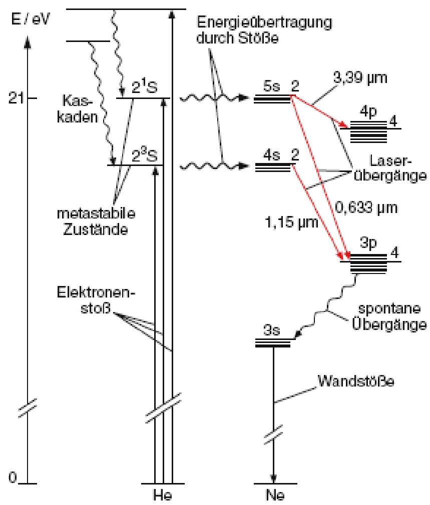

Figure 3: The four-level system of the HeNe-Laser (from [2])

3.1 Transversal modes

Our target was to illustrate the different TEM_{nm} modes of a laser beam using a cross for the TEM_{11} mode and a hair for the TEM_{10}/TEM_{01} mode. We expected to see spots like these:

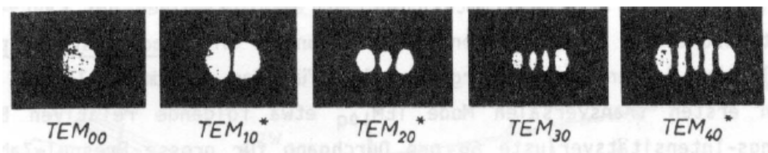

Figure 4: Look of different *TEM* modes (from [3])

Unfortunately the hair as well as the cross seemed to absorb too much of the laser's power. This terminated the laser's beam and thus gave us no result. According to this we can see that the gain is just too low to support other modes - and that the TEM_{00} mode is the most efficient one.

3.2 Determination of the laser's wave-length

We used a diffraction grating to produce a diffraction image on a screen. The diffraction grating constant was $g = 1$ mm. In order to get the wave-length at the k-th maximum we had to use the formula

$$k\lambda = g(\sin\alpha - \sin\theta_k), \tag{1}$$

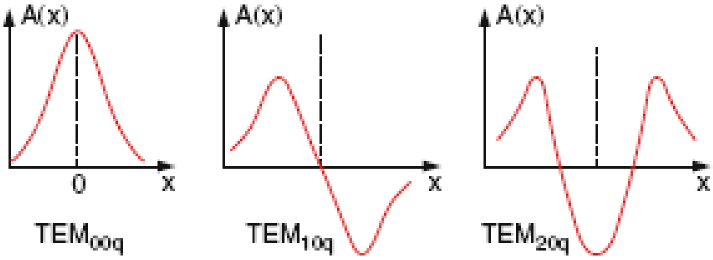

Figure 5: Amplitude distribution in various *TEM* modes (from [2])

with α being half the angle of the center maximum to the not diffracted beam (compare to figure (6)). Thus we get for α

$$\alpha = \frac{\pi}{2} - \arctan\left(\frac{x_0}{l}\right), \tag{2}$$

with x_0 being half of the difference between the center maximum and the not diffracted beam and l being the distance to the diffraction grating. For θ_k

$$\theta_k = \frac{\pi}{2} - \arctan\left(\frac{x_k}{l}\right), \tag{3}$$

with x_k being half of the difference of the k-th maximum to the not diffracted beam. Inserting (3) and (2) into (1) we gather

$$\lambda_k = \frac{g}{k}\left(\cos\left(\arctan\left(\frac{x_0}{l}\right)\right) - \cos\left(\arctan\left(\frac{x_k}{l}\right)\right)\right).$$

Through measuring we received the data (Δx_k being the difference of x_k to x_0) displayed in table (1). Taking the average value gives us the result of

$$\lambda = (646 \pm 40)\ \text{nm}.$$

This is a bit larger than the value which can be found in common literatur of 632.8 nm, but in good agreement with it, if we take the error of our measurement into account.

3.3 Amplification factor

Solar cell In order to retrieve information about the amplification factor of the active medium we used a solar cell and different absorbing glas plates. The

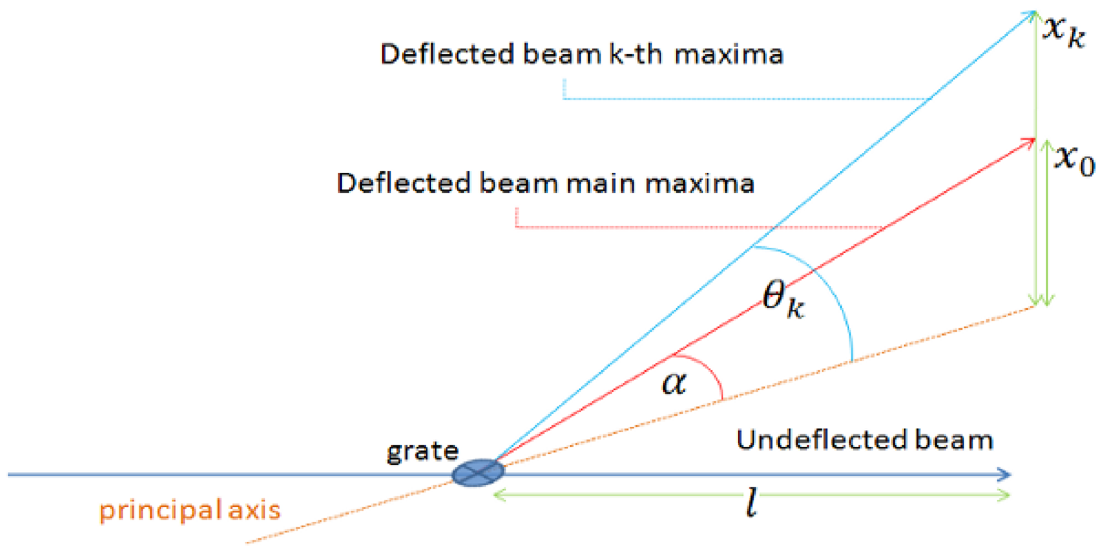

Figure 6: Sketch of the measurement

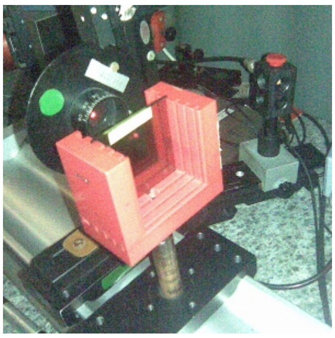

Figure 7: Experimental setup for measuring the amplification factor

charastic chart of each plate was used in order to find out the transmission coefficient.

It was possible to gather the data shown in table (2). We expected to find a linear dependence, because of the solar cell being linear in the relevant region. Besides a small error in the lower regions our expectations were confirmed. This exception can be due to unexact tables or thicknesses of the given glasses.

Measurement of the amplification factor The transmitted intensity I is theoreticaly given by the so called Airy-function:

k	Δx_k [cm]	λ_k [nm]
1	1.1	606
2	2.1	634
3	2.95	638
4	3.77	653
5	4.48	654
6	5.15	657
7	5.8	662
8	6.3	650
9	6.88	654
10	7.4	654
11	7.9	653

Table 1: Measured wavelengths

Type	d [mm]	I [mA]	T
no glas	0	0.064	1
NG5	1	0.031	0.5
NG5	2	0.019	0.25
NG4	2	0.006	0.1
NG4	1	0.017	0.33
NG11	2	0.034	0.55
hand	∞	0	0

Table 2: Amplification factor

$$\begin{aligned} I &= I_0 \frac{T^2}{(1-R^2)^2} A(\delta), \qquad R = \left(\frac{n-1}{n+1}\right)^2 \approx 0.04 \\ A(\delta) &= \frac{1}{1+F\sin^2(\frac{\delta}{2})}, \qquad F = \frac{4R}{(1-R)^2} \end{aligned}$$

with T bing the transmissivity, R the reflectivity, $T = 1 - R$, $n \approx 1.5$ is the refraction index o f the glas, F the finesse,

$$\delta = \frac{4\pi n}{\lambda}\cos(\Theta).$$

Θ is the angle between the optical axis and the vector orthogonal to the surface of the glas plate. If one inserted the plate theoretically one should get a periodic increase and decrease of the intensity while changing Θ. If one then plots the intensity as a function of $\cos(\Theta)$ one could scale the axis of

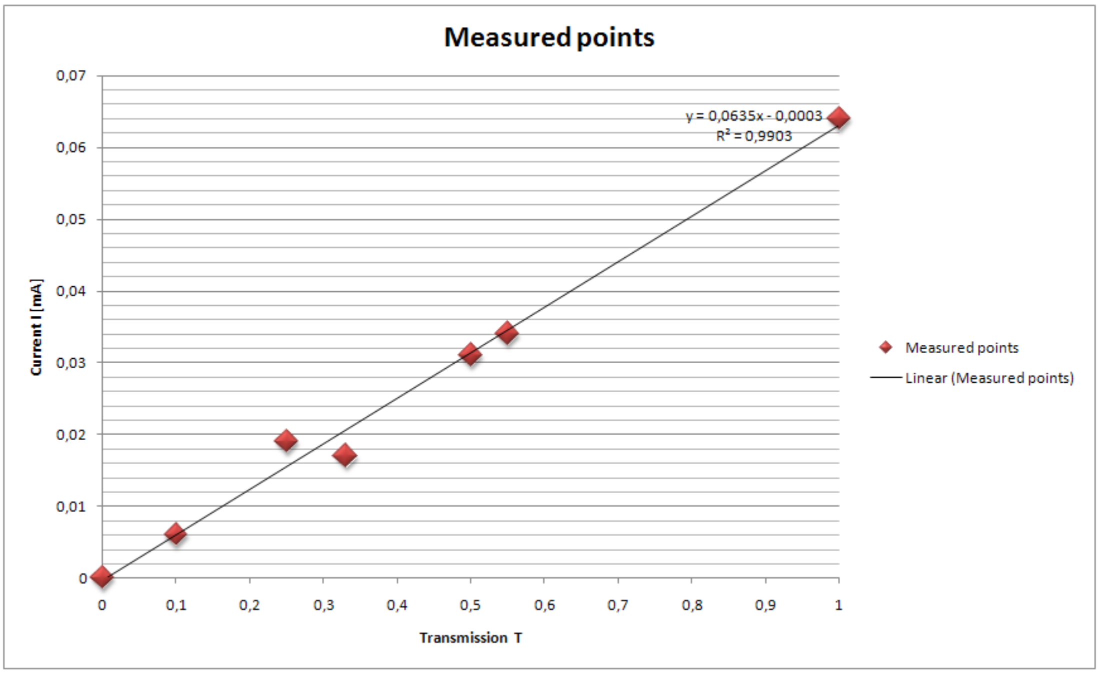

Figure 8: Measured current-transmission dependence

$$\delta \propto \cos(\Theta),$$

because of the 2π-periodicity of the Airy-function. After that one could simply determine the value of δ_0 from the graph. In our case the losses due to inserting the glas plate were too high, so that the lasing condition could not be satisfied anymore. Therefore we couldn't poperly perform the proposed measurements.

3.4 Measurement of the speed of light

The next part of our experiment was about measuring the speed of light by measuring the free spectral range between two longitudinal modes of the laser. We used a spectrum analyser and measured the frequency of the first peak, which is the first frequency wich satisfies the condition for longitudianl modes, at two different resonator lengths. The measured values can be found in table (3).

Since we have a very small error in the frequency (about 0.1 MHz for each measure - which is smaller than 0.1%) we can forget about this error and

Resonator length L [m]	Frequency $\Delta\nu$ [MHz]
0.89	168.9
0.85	187.9
0.74	219.1
0.59	261.2

Table 3: Frequencies at different resonator lengths

concentrate on the error in length. We get a total error of 6% in length and thus in the total calculation of the speed of light using the formula

$$c = \frac{2\Delta L}{\frac{1}{\Delta\nu_{L+\Delta L}} - \frac{1}{\Delta\nu_L}},$$

where L is the length of the resonator and ν is the frequency. Taking the average of all possible combinations we measured the speed of light at

$$c = (2.9 \pm 0.2) \cdot 10^8 \text{ m/s}. \tag{4}$$

We can use this value to calculate the length of the resonator L. We already know that we have standing waves fulfilling the condition

$$k\lambda = 2L, \qquad \Rightarrow \nu = \frac{kc}{2L}.$$

The frequency difference between two modes $k-1$ and k can then be written as

$$\Delta\nu_{k-1,k} = \frac{c}{2L}, \qquad L = \frac{c}{2\Delta\nu_{k-1,k}}. \tag{5}$$

Thus we are able to get unknown resonator lengths just using the measured speed of light (4), the $\Delta\nu$ frequency and equation (5).

3.5 Modulation of the laser

For the last part of the experiment we tried to transmit information by modulating the properties of the laser beam. We used a Faraday-Rotator which rotates proportial to magnetic flux. This can be described by

$$\beta = VBd,$$

where V is the material dependent Verdett constant and d the thickness of the Faraday-Rotator. β is the angle of rotation. After the laser's beam passed the Faraday-Rotator it ends in a photo transistor which is plugged into a speaker. The Faraday-Rotator was driven by a frequency generator which produced a sinus-like signal in hearable frequency range. After minimizing the background noise, we were able to hear the corresponding output on the speaker when changing the generated frequency.
We tried to transmit the signal through the given fiber, too, but one of the two available fibers was broken and the other one had too great losses for our purposes. We could only see a very weak transmission of the laser beam.

4 Conclusion

Though many parts of the experiment were not working or not working as intended the experiment was very interesting. We have seen that the laser is binary device - only working or not working. Modifying the lasers modulation and testing various settings gave us a deeper insight on the sensitive and fragile operation of the laser. Overall we got the following results:

- We were able to show that the resonator length has an enourmous impact on the laser's behaviour.
- We could clearly see that the TEM_{00} mode is the most effective laser operation mode. We were not able to get other modes due to their dramatically increased loss factors.
- The wavelength was determined to be around (646 ± 40) nm which is in good accordance to the common literature value of 633 nm.
- The linear depence of the characteristic line of the solar cell was confirmed.
- The measured speed of light was $(2.9 \pm 0.2) \cdot 10^8$ m/s which is near to the real value of $c = 2,99792458 \cdot 10^8$ m/s.
- The Faraday-Rotator is an elegant way to manipulate the polarisation of the laser's beam. We saw the disadvantages of modern telecommunication cables.

List of Tables

List of Figures

References

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